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## USE OF HARMONIC MEAN TO FIND SOLUTION OF TRANSPORTATION PROBLEM

Raghubir Dyal\*

\*Department of Mathematics, Govt. College, Sri Muktsar Sahib, INDIA

### Abstract

In this paper we have used Harmonic mean to find penalties and then made allocations. Proposed approach gives better initial basic feasible solution than Vogels Approximation Method. The methodology has been illustrated by an example.

**Keywords:** Transportation Problem, Initial Basic Feasible Solution, Optimal Solution.

## I. INTRODUCTION

A certain class of linear programming problem known as transportation problems arises very frequently in practical applications. The classical transportation problem received its name because it arises naturally in the contacts of determining optimum shipping pattern. For example: A product may be transported from factories to retail stores. The factories are the sources and the store is the destinations. The amount of products that is available is known and the demands are also known. The problem is that different legs of the network joining the sources to the destination have different costs associated with them. The aim is to find the minimum cost routing of products from the supply point to the destination. The general transportation problem can be formulated as: A product is available at each of  $m$  origin and it is required that the given quantities of the product be shipped to each of  $n$  destinations. The minimum cost of shipping a unit of the product from any origin to any destination is known. The shipping schedule which minimizes the total cost of shipment is to be determined. The problem can be formulated as:

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^n x_{ij} = a_i, i = 1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, \dots, n$$

$x_{ij} \geq 0$ , for all  $i, j$ .

For each supply point  $i$ , ( $i = 1, 2, \dots, m$ ) and demand point  $j$ , ( $j = 1, 2, \dots, n$ )

$c_{ij}$  = unit transportation cost from  $i^{\text{th}}$  source to  $j^{\text{th}}$  destination

$x_{ij}$  = amount of homogeneous product transported from  $i^{\text{th}}$  source to  $j^{\text{th}}$  destination

$a_i$  = amount of supply at  $i^{\text{th}}$  source.

$b_j$  = amount of demand at  $j^{\text{th}}$  destination.

where  $a_i$  and  $b_j$  are given non-negative numbers and assumed that total supply is equal to total demand, i.e.  $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$ , then transportation problem is called balanced otherwise it is called unbalanced. The aim is to minimize the objective function satisfying the above mentioned constraints. In the classical transportation problem of linear programming, the traditional objective is one of minimizing the total cost.

Because of the special structure of the transportation model, the problem can also be represented as Table 1.

**Table 1: Tabular representation of model (a)**

Destination → source ↓	D <sub>1</sub>	D <sub>2</sub>	...	D <sub>n</sub>	supply( $a_i$ )
S <sub>1</sub>	c <sub>11</sub>	c <sub>12</sub>	...	c <sub>1n</sub>	a <sub>1</sub>
S <sub>2</sub>	c <sub>21</sub>	c <sub>22</sub>	...	c <sub>2n</sub>	a <sub>2</sub>
⋮	⋮	⋮	...	⋮	⋮
S <sub>m</sub>	c <sub>m1</sub>	c <sub>m2</sub>	...	c <sub>mn</sub>	a <sub>m</sub>
Demand( $b_j$ )	b <sub>1</sub>	b <sub>2</sub>	...	b <sub>n</sub>	

**Algorithm of Proposed Method**

To proceed with proposed method the given steps are followed:

- step 1. Represent the given TP into the form of cost matrix as Table 1.
- step 2. Balance the given TP, if it is not balanced by adding dummy row/column according to requirement of supply/demand.
- step 3. Find harmonic mean of each row and each column of the given TP.
- step 4. Select the row or column which has highest harmonic mean and make maximum possible allocation in the cell having minimum cost in the selected row or column.
- step 5. Cross the row or column for which demand or supply is satisfied
- step 6. Repeat the step 4 and step 5 until whole demand and supply not satisfied.
- step 7. Calculate the transportation cost by adding the multiplication of all allocations and corresponding cost.

**II. NUMERICAL EXAMPLES**

Numerical example: Mr. Rajiv has three factories manufacturing containers of vary large capacity. Mr. requires containers at four different destinations. The transportation cost, supply and demand are shown in the Table. Input data and optimal solution obtained by applying Harmonic Mean for example is given in table 2

**Table 2: Input data and optimal solution**

Ex.	Input Data	Obtained Allocations by Harmonic Mean Method	Obtained Cost
1	$[c_{ij}]_{3 \times 3} = [6 \ 10 \ 14 ; 12 \ 19 \ 21; 15 \ 14 \ 17; ]$ ; $[a_i]_{3 \times 1} = [50, 50, 50 ]$ ; $[b_j]_{1 \times 3} = [30, 40, 55]$	$x_{11} = 30$ $x_{12} = 20$ , $x_{23} = 25$ , $x_{32} = 20$ , $x_{33} = 30$ ,	1695

**Table 3: Input data and optimal solution**

Ex.	Input Data	Obtained Allocations by VAM	Obtained Cost
1	$[c_{ij}]_{3 \times 3} = [6 \ 10 \ 14; 12 \ 19 \ 21; 15 \ 14 \ 17; ]$ ; $[a_i]_{3 \times 1} = [50, 50, 50 ]$ ; $[b_j]_{1 \times 3} = [30, 40, 55]$	$x_{12} = 40$ $x_{13} = 10$ , $x_{21} = 30$ , $x_{23} = 20$ , $x_{33} = 25$ ,	1745

After applying Modi Method optimal solution obtained is 1650, which is nearer to proposed method. Thus our approach is better than VAM

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